Mathematics Classroom Observation Protocol for Practices: Descriptors Manual

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The Mathematics Classroom Observation Protocol for Practices (MCOP^2) is a K-16 mathematics classroom instrument designed to measure the degree of alignment of the mathematics classroom with the various standards set out by the corresponding national organization that focus on conceptual understanding in the mathematics classroom including:

- Common Core State Standards in Mathematics: Standards for Mathematical Practice **(National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010)**,
- Mathematical Association of America (MAA): CUPM Curriculum Guide **(Barker, et al., 2004)**,
- American Mathematical Association of Two-Year Colleges (AMATYC): "Crossroads" **(AMATYC, 1995)** and "Beyond Crossroads" **(AMATYC, 2006)**, and
- National Council of Teachers of Mathematics (NCTM): Process Standards **(NCTM, 2000)**.

Recommended Uses

The MCOP^2 form is designed to measure the activities occurring in a mathematics classroom during a single lesson. However, if one desires to measure the overall activities of a class, the form should be used to measure at least three different class settings. **An important item to remember is that while all of the items in the observation protocol are desired qualities of a mathematics classroom, not all of them are expected to be observed during a single lesson. It is expected that this instrument be used in a formative manner on single observations. Summatively, 3-6 observations are ideal in evaluating classroom instruction.**

The MCOP^2 form is not designed to be used during a single lesson or day to evaluate the teaching and learning atmosphere of the mathematics classroom.

When completing the MCOP^2 form, it is essential that the descriptors outlined in this manual are followed to maintain the validity and reliability of the instrument.

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How to Score

The MCOP^2 measures two distinct factors of Teacher Facilitation and Student Engagement through two subscales of 9 items each. (The MCOP^2 is not designed to get a single score of a classroom.)

The Teacher Facilitation subscale (Cronbach alpha of 0.850) measures the role of the teacher as the one who provides structure for the lesson and guides the problem solving process and classroom discourse. To calculate the score for the Teacher Facilitation subscale, one would add the scores for items 4, 6-11, 13, and 16.

The Student Engagement subscale (Cronbach alpha of 0.897) measures the role of the student in the classroom and their engagement in the learning process. To calculate the score for the Student Engagement subscale, one would add the scores for items 1-5 and 12-15.

Item	Student Engagement	Teacher Facilitation
1	X	
2	X	
3	X	
4	X	X
5	X	
6		X
7		X
8		X
9		X
10		X
11		X
12	X	
13	X	X
14	X	
15	X	
16		X

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1) Students engaged in exploration/investigation/problem solving.

The role of exploration, investigation, and problem solving is central in teaching mathematics as a process. In order for students to develop a flexible use of mathematics, they must be allowed to engage in exploration, investigation, and/or problem solving activities which go beyond following procedures presented by the teacher. Furthermore, problem solving can be developed as a valuable skill in itself (Barker, et al., 2004) and a way of thinking (NCTM, 1989), rather than just as the means to an end of finding the correct answer. Student exploration may also promote a stance of mathematics as a discipline that can be explored, reasoned about, connected to other subjects, and one that 'makes sense' (Barker, et al., 2004).

> Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010).

If students are following a procedure established by the teacher, then it does not count as exploration/investigation/problem solving. Instead, students should be determining their own solution pathway without necessarily knowing that the path will lead to the desired result.

Score	Description
3	Students regularly engaged in exploration, investigation, or problem solving. Over the course of the lesson, the majority of the students engaged in exploration/investigation/problem solving.
2	Students sometimes engaged in exploration, investigation, or problem solving. Several students engaged in problem solving, but not the majority of the class.
1	Students seldom engaged in exploration, investigation, or problem solving. This tended to be limited to one or a few students engaged in problem solving while other students watched but did not actively participate.
0	Students did not engage in exploration, investigation, or problem solving. There were either no instances of investigation or problem solving, or the instances were carried out by the teacher without active participation by any students.

2) **Students used a variety of means (models, drawings, graphs, concrete materials, manipulatives, etc.) to represent concepts.**

In mathematics instruction it is common for the teacher to use various representations (models, drawings, graphs, concrete materials, manipulatives, graphing calculators, compass & protractor, i.e. tools for the mathematics classroom) to focus students' thinking on and develop their conceptions of a mathematical concept. It is also important for students to interact with and develop representations of mathematical concepts and not merely observe the teacher presenting such representations. Thus, this item is concerned with whether the students use representations to represent mathematical concepts. The representations can be student generated (a drawing or a graph) or provided by the teacher (manipulatives or a table), but it is the students that must then use the representation. Just because there is a representation in a lesson, if it is only used by the teacher while students watch (such as a graph on a PowerPoint slide), it is not considered to be used by students unless the students manipulate and interact with the representation.

Students' notes can count as a type of representation if the students themselves offer some sort of input. For instance, if a student corrects a teacher's mistake in a problem he or she is copying down then the notes are actually being manipulated by a student and should therefore count as a type of representation.

Score	**Description**
3	The students manipulated or generated two or more representations to represent the same concept, and the connections across the various representations, relationships of the representations to the underlying concept, and applicability or the efficiency of the representations were explicitly discussed by the teacher or students, as appropriate.
2	The students manipulated or generated two or more representations to represent the same concept, but the connections across the various representations, relationships of the representations to the underlying concept, and applicability or the efficiency of the representations were **not** explicitly discussed by the teacher or students.
1	The students manipulated or generated one representation of a concept.
0	There were either no representations included in the lesson, or representations were included but were exclusively manipulated and used by the teacher. If the students only watched the teacher manipulate the representation and did not interact with a representation themselves, it should be scored a 0.

3) Students were engaged in mathematical activities.

This item is concerned with the extent of student engagement in activities that are mathematical. Students are considered to be engaged in a mathematical activity when they are investigating, problem solving, reasoning, modeling, calculating, or justifying (each of these could be written or verbal).

Note "most of the students" in an undergraduate mathematics classroom is accepted here to mean more than one-third of the students in the classroom were engaged in mathematical activity, while in a K-12 mathematics classroom it means more than one-half.

It is important to note that one should only focus on what actually happens—not what the teacher assigns watching for students who are off-task.

Score	Description
3	Most of the students spend two-thirds or more of the lesson engaged in mathematical activity at the appropriate level for the class. It does not matter if it is one prolonged activity or several shorter activities. (Note that listening and taking notes does not qualify as a mathematical activity unless the students are filling in the notes and interacting with the lesson mathematically.)
2	Most of the students spend more than one-quarter but less than two-thirds of the lesson engaged in appropriate level mathematical activity. It does not matter if it is one prolonged activity or several shorter activities.
1	Most of the students spend less than one-quarter of the lesson engaged in appropriate level mathematical activity. There is at least one instance of students' mathematical engagement.
0	Most of the students are not engaged in appropriate level mathematical activity. This could be because they are never asked to engage in any activity and spend the lesson listening to the teacher and/or copying notes, or it could be because the activity they are engaged in is not mathematical – such as a coloring activity.

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4) Students critically assessed mathematical strategies.

In order for students to flexibly use mathematical strategies, they must develop ways to consider the appropriateness of a strategy for a given problem, task, or situation. This is because not all strategies will work on all problems, and furthermore the efficiency of the strategy for the given context needs to be considered. For students to make such distinctions it is important that they have opportunities to assess mathematical strategies so that they learn to reason not only about content but also about process. This item is concerned with *students* critically assessing strategies, which is more than listening to the teacher critically assessing strategies or asking peers how they solved a task. Examples of critical assessment include students offering a more efficient strategy, asking "why" a strategy was used, comparing/contrasting multiple strategies, discussing the generalizability of a strategy, or discussing the efficiency of different ways of solving a problem (e.g. the selection appropriate tools if needed).

To score high on this item it is the students who must be engaged in the critical assessment, not only the teacher.

Score	Description
3	More than half of the students critically assessed mathematical strategies. This could have happened in a variety of scenarios, including in the context of partner work, small group work, or a student making a comment during direct instruction or individually to the teacher.
2	At least two but less than half of the students critically assessed mathematical strategies. This could have happened in a variety of scenarios, including in the context of partner work, small group work, or a student making a comment during direct instruction or individually to the teacher.
1	An individual student critically assessed mathematical strategies. This could have happened in a variety of scenarios, including in the context of partner work, small group work, or a student making a comment during direct instruction or individually to the teacher. The critical assessment was limited to one student.
0	Students did not critically assess mathematical strategies. This could happen for one of three reasons: 1) No strategies were used during the lesson; 2) Strategies were used but were not discussed critically. For example, the strategy may have been discussed in terms of how it was used on the specific problem, but its use was not discussed more generally; 3) Strategies were discussed critically by the teacher but this amounted to the teacher telling the students about the strategy(ies), and students did not actively participate.

5) **Students persevered in problem solving.**

One of the *Standards for Mathematical Practice* (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010) is that students will persevere in problem solving. Student perseverance in problem solving is also addressed in the Mathematical Association of America's Committee on the Undergraduate Program in Mathematics Curriculum Guide (Barker, et al., 2004):

> Every course should incorporate activities that will help all students…approach problem solving with a willingness to try multiple approaches, persist in the face of difficulties, assess the correctness of solutions, explore examples, pose questions, and devise and test conjectures.

Perseverance is more than just completion or compliance for an assignment. It should involve students overcoming a road block in the problem solving process.

Score	Description
3	Students exhibited a strong amount of perseverance in problem solving. The majority of students looked for entry points and solution paths, monitored and evaluated progress, and changed course if necessary. When confronted with an obstacle (such as how to begin or what to do next), the majority of students continued to use resources (physical tools as well as mental reasoning) to continue to work on the problem.
2	Students exhibited some perseverance in problem solving. Half of students looked for entry points and solution paths, monitored and evaluated progress, and changed course if necessary. When confronted with an obstacle (such as how to begin or what to do next), half of students continued to use resources (physical tools as well as mental reasoning) to continue to work on the problem.
1	Students exhibited minimal perseverance in problem solving. At least one student but less than half of students looked for entry points and solution paths, monitored and evaluated progress, and changed course if necessary. When confronted with an obstacle (such as how to begin or what to do next), at least one student but less than half of students continued to use resources (physical tools as well as mental reasoning) to continue to work on the problem. There must be a road block to score above a 0.
0	Students did not persevere in problem solving. This could be because there was no student problem solving in the lesson, or because when presented with a problem solving situation no students persevered. That is to say, all students either could not figure out how to get started on a problem, or when they confronted an obstacle in their strategy they stopped working.

6) **The lesson involved fundamental concepts of the subject to promote relational/conceptual understanding.**

Relational/conceptual understanding is "knowing both what to do and why" (Skemp, 1976). This is in contrast to a procedural understanding as being able to compute certain mathematical activities, but not understanding how the computation works or when one would need to use such a computation and what the answer would mean.

According to the NCTM (2006), certain topics are core to the mathematics learned at each grade level and can form the backbone of the K-8 curriculum. The NCTM extended this concept to the high school level with an emphasis on using these fundamental concepts to make sense of mathematics and deepen students' relational and conceptual understanding (Martin, et al., 2009). Similar to the NCTM's guidelines for middle school and high school mathematics lessons, at the undergraduate level the Mathematical Association of America has recommendations in the Committee on the Undergraduate Program in Mathematics Curriculum Guide (Barker, et al., 2004) for departments, programs, and all courses to promote relational/conceptual understanding for both mathematics majors and non-mathematics majors.

Score	Description
3	The lesson includes fundamental concepts or critical areas of the course, as described by the appropriate standards, and the teacher/lesson uses these concepts to build relational/conceptual understanding of the students with a focus on the "why" behind any procedures included.
2	The lesson includes fundamental concepts or critical areas of the course, as described by the appropriate standards, but the teacher/lesson misses several opportunities to use these concepts to build relational/conceptual understanding of the students with a focus on the "why" behind any procedures included.
1	The lesson mentions some fundamental concepts of mathematics, but does not use these concepts to develop the relational/conceptual understanding of the students. For example, in a lesson on the slope of the line, the teacher mentions that it is related to ratios, but does not help the students to understand how it is related and how that can help them to better understand the concept of slope.
0	The lesson consists of several mathematical problems with no guidance to make connections with any of the fundamental mathematical concepts. This usually occurs with a teacher focusing on procedure of solving certain types of problems without the students understanding the "why" behind the procedures.

7) The lesson promoted modeling with mathematics.

Following the "Standards for Mathematical Practice" from the Common Core State Standards (2010) and the recommendations from the MAA's CUPM Curriculum Guide (Barker, et al., 2004), this item describes lessons that help students to "apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another" (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010).

In an undergraduate classroom, a lesson that promotes modeling might use "radiocarbon dating to illustrate how an initial value problem (IVP) can model a real world situation, and the solution of the IVP then yields obviously useful and interesting results" or "a simple system of differential equations to predict the cyclical population swings in a predator-prey relationship" or even "how modular arithmetic is used in cryptography and the transmission of encoded information" (Barker, et al., 2004).

Score	Description
3	Modeling (using a mathematical model to describe a real-world situation) is an integral component of the lesson with students engaged in the modeling cycle (as described in the Common Core State Standards).
2	Modeling is a major component, but the modeling has been turned into a procedure (i.e. a group of word problems that all follow the same form and the teacher has guided the students to find the key pieces of information and how to plug them into a procedure.); **or** modeling is not a major component, but the students engage in a modeling activity that fits within the corresponding standard of mathematical practice.
1	The teacher describes some type of mathematical model to describe real-world situations, but the students do not engage in activities related to using mathematical models.
0	The lesson does not include any modeling with mathematics.

8) **The lesson provided opportunities to examine mathematical structure. (Symbolic notation, patterns, generalizations, conjectures, etc.)**

Following some of the "Standards for Mathematical Practice" (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010) and the recommendations in the MAA's CUPM Curriculum Guide (Barker, et al., 2004), lessons should include opportunities for students to contextualize and/or decontextualize in the process of solving quantitative problems, explore and make use of mathematical structure, or to use repeated reasoning to generalize certain categories of problems and their solutions.

Score	Description
3	The students have a sufficient amount of time and opportunity to look for and make use of mathematical structure or patterns.
2	Students are given some time to examine mathematical structure, but are not allowed adequate time or are given too much scaffolding so that they cannot fully understand the generalization.
1	Students are shown generalizations involving mathematical structure, but have little opportunity to discover these generalizations themselves or adequate time to understand the generalization.
0	Students are given no opportunities to explore or understand the mathematical structure of a situation.

9) The lesson included tasks that have multiple paths to a solution or multiple solutions.

As part of having students "make sense of problems and persevere in solving them" (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010), students must be encouraged to look for multiple methods of solving a problem and to deal with problems that have multiple solutions based upon various assumptions. Additionally, selected tasks with multiple paths to a solution or multiple solutions can increase the cognitive demand of the task for all students through the interaction of the teacher to ask questions of each student at their ability level (Stein & Smith, 1998). This flexibility, "switching (smoothly) between different strategies," and adaptivity, "selecting the most appropriate strategy" (Verschaffel, Luwel, Torbeyns, & Van Dooren, 2009) enables students to solve problems for which a solution path is not obvious.

Score	Description
3	A lesson which includes several tasks throughout; or a single task that takes up a large portion of the lesson; with multiple solutions and/or multiple paths to a solution and which increases the cognitive level of the task for different students.
2	Multiple solutions and/or multiple paths to a solution are a significant part of the lesson, but are not the primary focus, or are not explicitly encouraged; **or** more than one task has multiple solutions and/or multiple paths to a solution that are explicitly encouraged.
1	Multiple solutions and/or multiple paths minimally occur, and are not explicitly encouraged; **or** a single task has multiple solutions and/or multiple paths to a solution that are explicitly encouraged.
0	A lesson which focuses on a single procedure to solve certain types of problems and/or strongly discourages students from trying different techniques.

10) The lesson promoted precision of mathematical language.

This item follows the Standard of Mathematical Practice to "attend to precision". As such, "Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem" (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010).

This item also follows the MAA's CUPM Curriculum Guide recommendation to "develop mathematical thinking and communication skills" which states: "Students should read mathematics with understanding and communicate mathematical ideas with clarity and coherence through writing and speaking" (Barker, et al., 2004).

Whether the communication is verbal or written and originating in the teacher or a student, using precise mathematical language is important. While the teacher cannot control the language used by students, there should be evidence of expectations of the teacher upon the students related to communicating with precise mathematical language. For example, if the lesson is primarily students solving problems, a culture of precision of language should come through in how the students are communicating with one another, both verbal and written.

Score	Description
3	The teacher "attends to precision" in regards to communication during the lesson. The students also "attend to precision" in communication, or the teacher guides students to modify or adapt non-precise communication to improve precision.
2	The teachers "attends to precision" in all communication during the lesson, but the students are not always required to also do so.
1	The teacher makes a few incorrect statements or is sloppy about mathematical language, but generally uses correct mathematical terms.
0	The teacher makes repeated incorrect statements or incorrect names for mathematical objects instead of their accepted mathematical names.

11) The teacher's talk encouraged student thinking.

This item assesses how well the teacher's talk promotes a number of the mathematical practices. Specifically, the practices requiring students to be able to think, reason, argue, and critique during the study of mathematical concepts (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). Teachers can greatly impact the level of student thinking and discussion simply by what questions are asked of students. In line with Stein, et al. (2009), the cognitive task level should be maintained at a high level, i.e. procedures with connections and doing mathematics, while questions which are over-scaffolded, rhetorical, or cursory to the level of the students, would score a 1 or a 0.

Specifically about the teacher's talk, this item is referring to the content of the question or statements put forth in the classroom for students to reason and/or discuss. A well planned lesson may contain rich tasks for students to explore or problems to solve, but if the teacher's talk drops or removes student reasoning and problem solving, it has removed or reduced student thinking.

Score	Description
3	The teacher's talk focused on **high levels** of mathematical thinking. The teacher may ask lower level questions within the lesson, but this is not the focus of the practice. There are three possibilities for high levels of thinking: analysis, synthesis, and evaluation. ***Analysis**:* examines/ interprets the pattern, order or relationship of the mathematics; parts of the form of thinking. **Synthesis**: requires original, creative thinking. **Evaluation**: makes a judgment of good or bad, right or wrong, according to the standards he/she values.
2	The teacher's talk focused on **mid-levels** of mathematical thinking. **Interpretation**: discovers relationships among facts, generalizations, definitions, values and skills. **Application**: requires identification and selection and use of appropriate generalizations and skills
1	Teacher talk consists of **"lower order"** knowledge based questions and responses focusing on recall of facts. **Memory**: recalls or memorizes information. **Translation**: changes information into a different symbolic form or situation.
0	Any questions/ responses of the teacher related to mathematical ideas were rhetorical in that there was no expectation of a response from the students.

12) There were a high proportion of students talking related to mathematics.

The focus of this descriptor is on the proportion of students talking (frequency). The Standards for Mathematical Practice (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010) encourages students to be active in making conjectures, exploring the truth of those conjectures, and responding to the conjectures and reasoning of others. In a classroom dominated by only a few students, classroom discourse may appear to be high, but all students must be engaged.

Score	Description
3	More than three quarters of the students were talking related to the mathematics of the lesson at some point during the lesson.
2	More than half, but less than three quarters of the students were talking related to the mathematics of the lesson at some point during the lesson.
1	Less than half of the students were talking related to the mathematics of the lesson.
0	No students talked related to the mathematics of the lesson.

13) There was a climate of respect for what others had to say.

This item adheres to the expectation provided in the third Standard for Mathematical Practice, "Construct viable arguments and critique the reasoning of others." Given that practice, students are expected to communicate with each other as part of an effective classroom community. Effective communication means that students will listen, question, and critique; this is part of the discourse expected in a mathematics classroom (Sherin, Mendez, & Louis, 2004). This item also encompasses the literature on equity and mathematics in that all students have valuable ideas, strategies, and thinking to share within the mathematics classroom (Boaler, 2006). Equitable spaces include the interactions of students within a mathematical community that increase participation and engagement of all students and work to remove potential barriers (Diversity in Mathematics Education Center for Learning and Teaching, 2007; Gutierrez, 2007; Hiebert & Grouws, 2007; NCTM, 2000; Sherin, Mendez, & Louis, 2004; Yackel & Cobb, 1996). This means creating a climate of respect.

Score	Description
3	**Many** students are sharing, questioning, and commenting during the lesson, including their struggles. Students are also listening (active), clarifying, and recognizing the ideas of others.
2	The environment is such that **some** students are sharing, questioning, and commenting during the lesson, including their struggles. Most students listen.
1	Only a **few** share as called on by the teacher. The climate supports those who understand or who behave appropriately. **Or** Some students are sharing, questioning, or commenting during the lesson, but most students are actively listening to the communication.
0	No students shared ideas.

14) In general, the teacher provided wait-time.

The appropriate wait time must align with the question/task. In the elementary grades, a teacher may ask students to explain a situation that represents the expression 24*(1/2)*3. In middle school, the teacher may ask students to describe why the slope is positive. High school teachers may ask students to explain how linear and exponential functions are similar and different. In each instance, these questions/tasks are not simple yes/no answer and require wait time to provide an answer with meaning and understanding.

Simple Yes/No questions could be asked, but must be accompanied by an explanation. Simple skills or procedural problems should require explanations with the computation and/or procedures. If the class is dominated by rhetorical questions, a score of 0 or 1 is warranted. Even if rhetorical questions are asked, it is possible to score a 2 or 3 if there are questions asked sometimes or frequently that require students to reason, make sense, and articulate thoughtful responses.

Score	Description
3	The teacher **frequently** provided an ample amount of "think time" for the depth and complexity of a task or question posed by either the teacher or a student.
2	The teacher **sometimes** provided an ample amount of "think time" for the depth and complexity of a task or question posed by either the teacher or a student.
1	The teacher **rarely** provided an ample amount of "think time" for the depth and complexity of a task or question posed by either the teacher or a student.
0	The teacher **never** provided an ample amount of "think time" for the depth and complexity of a task or question posed by either the teacher or a student.

15) **Students were involved in the communication of their ideas to others (peer-to-peer).**

Both the National Council of Teachers of Mathematics and The Eight Standards for Mathematical Practices, expect teachers to create a mathematical community that includes dialogue around the mathematics content and learning. Students are expected to talk and participate in the discourse of the classroom (Manouchehri & St John, 2006). This item highlights the need for all students to be active participants in the classroom dialogue. Without teacher support and expectations, the classroom discourse can be monopolized or biased against certain populations (Mercer & Wegerif, 1999; Mercer, Wegerif, & Dawes, 1999; Rojas-Drummond & Mercer, 2003; Rojas-Drummond & Zapata, 2004).

This descriptor focuses on the amount of time students spend in communication with their peers at any level, including pairs, groups, informal settings, or whole class settings.

Score	**Description**
3	**Considerable** time **(more than half)** was spent with peer to peer dialog (pairs, groups, whole class) related to the communication of ideas, strategies and solution.
2	**Some** class time **(less than half, but more than just a few minutes)** was devoted to peer to peer (pairs, groups, whole class) conversations related to the mathematics.
1	The lesson was primarily teacher directed and little opportunities were available for peer to peer (pairs, groups, whole class) conversations. A few instances developed where this occurred during the lesson but only lasted less than 5 minutes.
0	No peer to peer (pairs, groups, whole class) conversations occurred during the lesson.

16) The teacher uses student questions/comments to enhance conceptual mathematical understanding.

Driscoll (1999; 2007) and Reys, et al. (2009) discuss how teacher questioning can build on student thinking to foster deeper mathematical thinking. In the elementary grades, students can make "over generalized" statements that have a correct nature about them. This is a teachable moment to use. A teacher can ask a question that has the student(s) reexamine their thoughts that would help simplify the over generalizing statement into precise understanding. Reys, et al. (2009) present a simple example, "Student: So every even number is composite. Teacher: Every even number? <Pause with wait time> What about 2?" The teacher's question stimulates further thought by the student. In secondary grades, Driscoll (1999) indicates that well-timed questions to students should help them shift or expand their thinking, or at least have students thinking about what is important to pay attention to during a lesson. When students are examining expressions, a teacher can ask questions to facilitate mathematical flexibility (Heinze, Star, & Verschaffel, 2009). For example, "What other ways can you write that expression to bring out the hidden meaning? How can you write the expression in terms of the important things you care about?"

Score	Description
3	The teacher **frequently** uses student questions/ comments to coach students, to facilitate conceptual understanding, and boost the conversation. The teacher sequences the student responses that will be displayed in an intentional order, and/or connects different students' responses to key mathematical ideas.
2	The teacher **sometimes** uses student questions/ comments to enhance conceptual understanding.
1	The teacher **rarely** uses student questions/ comments to enhance conceptual mathematical understanding. The focus is more on procedural knowledge of the task verses conceptual knowledge of the content.
0	The teacher **never** uses student questions/ comments to enhance conceptual mathematical understanding.

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